On the Analysis of Fully Cartesian Coordinates – A Comparison between a Reduced and Full-Defined Modelling Approach in Spatial Mechanisms

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EXTENDED ABSTRACT

1 Introduction

Different multibody formulations have been purposed for the study of mechanical systems, varying in the way the bodies are defined, the nature of the used coordinates and in the complexity of the constraint and governing equations. In a recent work, the authors presented an approach to model planar multibody systems using fully Cartesian coordinates (FCC) [1]. As in the natural coordinates formulation, the position and orientation of each body is defined resorting solely to the Cartesian coordinates of points and unit vectors, generating kinematic constraints that have a quadratic or linear dependency on the system generalized coordinates. However, and contrarily to the natural coordinates, the proposed formulation adopts the concept of a generic rigid body, which simplifies the modelling procedure and approximates it from the one used in Cartesian Coordinates.

Despite sharing some of the advantages of these two global formulations, the FCC formulation with a generic rigid body tends to generate more generalized coordinates, particularly, when used to model spatial mechanisms. Considering a full definition in 3D, each rigid body is described using one point and three non-coplanar vectors, generating 12 generalized coordinates, and requiring six constraint equations of rigid body type. An alternative approach to decrease the number of generalized coordinates is to use a reduced definition of the rigid body. In this approach, the body is defined using only one point and two non-collinear vectors, generating a total of nine generalized coordinates. However, this approach requires further computations that can compromise the computational efficiency.

Hence, this work expands the formulation presented in [1] to the analysis of spatial systems and evaluates the computational differences between using a full-defined and a reduced modelling approach of the rigid bodies, considering the simulation of four classical benchmark problems, namely the fall of a constrained rigid body, a double-pendulum, a double four-bar linkage and a spatial slider-crank [2]. Moreover, an analysis of the influence of increasing of the number of rigid bodies is performed by evaluating the differences in the computational performance of a *n*-four bar mechanism with a variable number of segments.

2 Methods

In the FCC formulation, the generic rigid body, in its fully-defined form, is modelled with a predetermined structure, composed of one point ($\mathbf{r}_{O_i}$) and three unit vectors ($\mathbf{u}_i$, $\mathbf{v}_i$, $\mathbf{w}_i$):

$$\mathbf{q}_i = \left\{\mathbf{r}_{O_i}^T \quad \mathbf{u}_i^T \quad \mathbf{v}_i^T \quad \mathbf{w}_i^T\right\}^T \tag{1}$$

where ($\mathbf{q}_i$) represents the generalized coordinates vector for a given rigid body *i*. Knowing that from two non-collinear vectors is possible to define a vector basis in 3D, the definition of the generic rigid body can be simplified, such as:

$$\mathbf{q}_i = \left\{\mathbf{r}_{O_i}^T \quad \mathbf{u}_i^T \quad \mathbf{v}_i^T\right\}^T \tag{2}$$

with:

$$\mathbf{q}_{3v_i} = \left\{\mathbf{r}_{O_i}^T \quad \mathbf{u}_i^T \quad \mathbf{v}_i^T \quad \mathbf{w}_i^{*T}\right\}^T = \mathbf{V}_i^{\mathbf{q}_{3v}}\mathbf{q}_i \tag{3}$$

where $\mathbf{q}_{3v_i}$ is the full-defined equivalent vector of the reduced rigid body i and $\mathbf{V}_i^{\mathbf{q}_{3v}}$ a transformation matrix that converts the reduced set of generalized coordinates of the rigid body in its full-defined equivalent form. Besides allowing for the reduction of the number of generalized coordinates, this modelling approach allows also for the reduction of the number of kinematic constraints of rigid body type. However, describing the kinematics of the system in terms of the full-defined equivalent vector, increases the order of the kinematic constraint equations, as the transformation matrix **V** presents an explicit dependency on the system generalized coordinates. Moreover, this procedure generates mass matrices and velocity-dependent inertial terms, which are dependent on the system state, thus requiring their update each time the equations of motion (EoM) are evaluated.

For assessing the accuracy of the spatial formulation and for comparing the differences in the computational performance between the two modelling approaches, the FCC formulation with a generic rigid body was implemented in an in-house software developed with MATLAB language and applied to the study of a set of benchmark problems. The kinematic outcomes were posteriorly compared with the analytical solutions or benchmark results available in the IFToMM database.

3 Results and Discussion

The expansion of the planar formulation presented in [1] for the spatial analysis of mechanical systems is detailed in this work. As the fundamental aspects of the formulation are maintained, the advantages reported for the planar case are still valid in the spatial version. Hence, the multibody system continues to be described using only Cartesian coordinates of points and vectors and the rigid bodies are defined with a predetermined kinematic structure. This approach makes the modeling of the rigid bodies independent of the system topology and generates kinematic constraints that present at the most a quadratic dependency on the generalized coordinates for the full-defined case. Similarly, the kinematics of any generic point or vector of the system can still be described using a set of constant transformation matrices (matrix **C**), simplifying the description of the system kinematics, the definition of the kinematic constraints and the system mass matrix, the application of external forces, among other advantages.

No convergence problems were found for the simulation of the benchmark problems with the two modelling approaches, presenting an excellent agreement with the reference data (see Fig. 1a). In the same way, no differences were found between the two cases, presenting similar kinematic patterns for the entire simulation. The comparison of the violation of the kinematic constraints show values of the same order for the two cases, and within the limits specified for the simulation (see Fig. 1 b). Similarly, the variation of the mechanical energy shows similar patterns between the two modelling approaches (see Fig. c).

As for the differences in the computational performance, the results show a pattern that varied with the complexity of the model under analysis. For the simplest case, the fall of a constrained rigid body, the results show that the reduced approach is approximately 40% faster. However, this difference decreased with the increase of the complexity of the models. For instance, the double four-bar linkage was nearly 15% faster on the full-defined approach. With the increase of the complexity of the model, by adding more four-bar linkages in the kinematic chain, the reduced approach becomes faster again, presenting a reduction in the computational time of approximately 21% for a *n* four-bar linkage with 20 vertical bars.

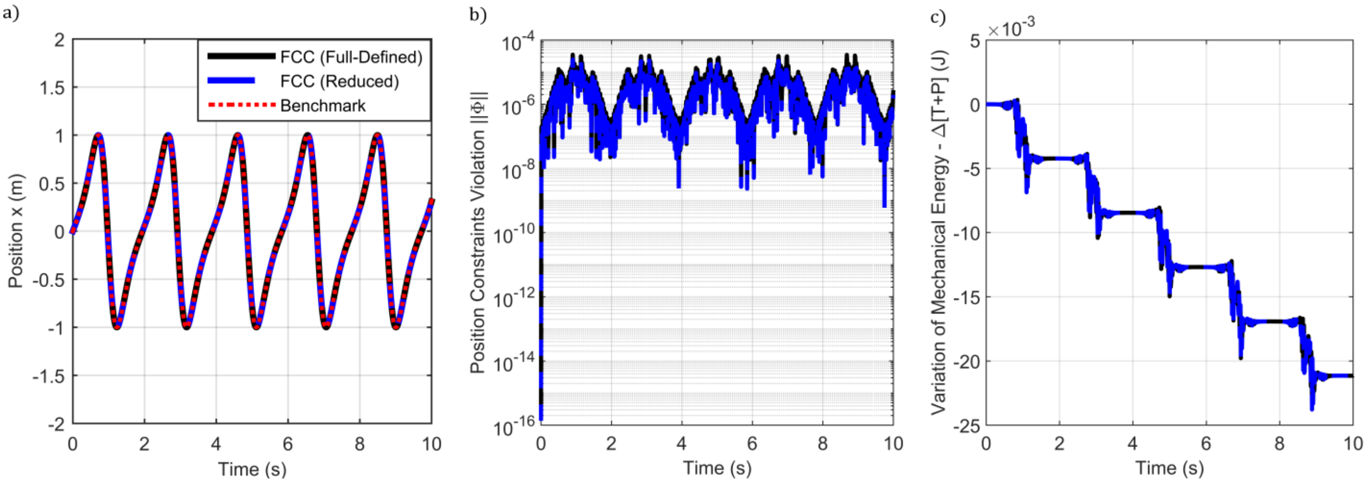

Figure 1: Comparison between the full-defined approach (black solid), reduced approach (dashed blue) and benchmark data (red dots; IFToMM Cuadrado) for the FD simulation of the double-four bar linkage: a) *x* coordinate of point P_2; b) Norm of the position constraints violation (logarithmic scale); c) Variation of the mechanical energy.

4 Conclusions

The expansion of the FCC formulation to the analysis of spatial multibody systems maintains the same theoretical fundaments which were defined for the planar case, i.e., the system is described using only Cartesian coordinates, the rigid bodies are defined with a fixed structure and the kinematic constraints present at the most a quadratic dependency on the generalized coordinates. No differences between the reduced and full-defined modelling approaches were found, presenting an excellent agreement with benchmark data. The results obtained for the computational times show that for the more complex models, the reduced approach tends to be faster, despite requiring more calculations every time the EoM are evaluated. Moreover, the simplicity in the modelling and implementation procedures are maintained in the spatial case, making the FCC formulation with a generic rigid body suitable for the teaching of courses related with the topics of multibody dynamics and biomechanics in higher education.

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